Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t)\,dt = 171.813$ mm

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{S(60) - S(0)}{60} = 2.863$ or 2.864 mm/day

1 : answer

(c) $V(t) = 100\pi S(t)$
$V'(7) = 100\pi S'(7) = 602.218$

The volume of water in the can is increasing at a rate of $602.218\ \text{mm}^3/\text{day}$.

$2 : \begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t, $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

$2 : \begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

1 : units in (b) or (c)

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

1(a). $\int_0^{60} 2\sin(0.03t)+1.5\, dt$

≈ 171.813 mm

Work for problem 1(b)

$S(60) = \int_0^{60} 2\sin(0.03t)+1.5\, dt$

$= 171.83$ mm.

$\dfrac{S(60)-S(0)}{60 \text{ days}}$ average rate of change in height.

$\dfrac{171.813 - 0 \text{ mm}}{60 \text{ days}} \approx 2.864$ mm/day

Continue problem 1 on page 5.

Work for problem 1(c)

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi \cdot 100 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \pi \cdot 100 \cdot 1.917$

$\frac{dV}{dt} \approx 602.218 \text{ mm}^3/\text{day}$

At $t=7$ $\frac{dh}{dt} = 2 \cdot \sin(0.03 \cdot 7) + 1.5$

≈ 1.917

Work for problem 1(d)

$M(t) = \frac{3}{400}t^3 - \frac{3}{40}t^2 + \frac{33}{40}t$

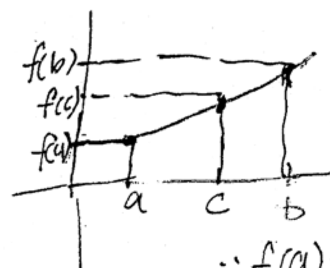
$M'(t) = 3 \cdot \frac{3}{400}t^2 - 2 \cdot \frac{3}{40}t + \frac{33}{40}$

$= \frac{9}{400}t^2 - \frac{3}{20}t + \frac{33}{40}$

$D(t) = \frac{9}{400}t^2 - \frac{3}{20}t + \frac{33}{40} - 2\sin(0.03t) + 1.5$

$D(0) = \frac{33}{40} - 1.5 = -0.675.$

$D(60) = 69.377$



$\because f(a) = -0.675 < 0$

$f(b) = 69.377 > 0$

$\therefore$ There exist a time c which f(c) or $D = 0$ $(M'(t) - S'(t) = 0)$

In order for both the cans' heights to change at the same rate $D(t) = 0 \rightarrow M'(t) - S'(t) = 0$.

$\therefore$ According to the IVT. If a function is continuous on the interval [a, b], and there exist corresponding value f(a) & f(b). in which f(a) < f(b), the c, a value in between (a, b) on the interval [a, b]. has a corresponding value in between f(a) & f(b).

GO ON TO THE NEXT PAGE.

 1B

CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

$$\int_0^{60} S'(t)\,dt$$

$$= \int_0^{60} 2\sin(0.03t)+1.5\,dt$$

$$= 171.813 \text{ millimeters}$$

Work for problem 1(b)

$$\frac{1}{60-0}\int_0^{60} S'(t)\,dt$$

$$= \frac{1}{60}\int_0^{60} 2\sin(0.03t)+1.5\,dt$$

$$= 2.86356 \text{ mill/day}$$

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Continue problem 1 on page 5.

Work for problem 1(c)

$V = \pi r^2 h = 100\pi h$

$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$= 100\pi\left(2\sin(0.03 \cdot 7) + 1.5\right)$

$= 602$ mil^3/day at $t=7$

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Do not write beyond this border.

Work for problem 1(d)

$M'(t) = \frac{1}{400}\left(9t^2 - 60t + 330\right)$

$D(t) = \left(\frac{1}{400}(9t^2 - 60t + 330)\right) -$ $2\sin(0.03t) + 1.5$

$M'(t) = S'(t)$ at $t = 11.8166$

GO ON TO THE NEXT PAGE.

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

$\int_0^{60} S'(t)\,dt = \int_0^{60} (2\sin(0.03t)+1.5)\,dt = 171.183$ millimeters

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Work for problem 1(b)

$S(t) = \int S'(t)\,dt = 1.5x - 66.67\cos(0.03t) + c$

$S(0) = 0$

$C = 66.67$, $0 = 1.5(0) - 66.67\cos(0.03(0)) + c$

$S(t) = 1.5x - 66.67\cos(0.03t) + 66.67$

average rate of change =

$\frac{S(60) - S(0)}{60} = \frac{171.818 - 0}{60} = 28.636$ mm/day

Continue problem 1 on page 5.

Work for problem 1(c)

$V = \pi r^2 h$

$V = \pi (10)^2 h$

$V = 100\pi h$

$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$S'(7) = \left.\frac{dh}{dt}\right|_{t=7}$

$S'(7) = 1.917$ mm/day

$\left.\frac{dV}{dt}\right|_{t=7} = 100\pi(1.917)$

$= 602.243$ mm^3/day

Work for problem 1(d)

$M'(t) = \frac{9x^2}{400} - \frac{3x}{20} + \frac{33}{40}$

$\int_0^{60} D(t)\,dt = \int_0^{60} M'(t) - S'(t)\ dt$

$\int_0^{60} D(t)\,dt = (60-0)\,D(c)$

$D(c) = 20.4614$

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GO ON TO THE NEXT PAGE.

Question 1

Sample: 1A
Score: 9

The student earned all 9 points. In part (d) the student considers $D(0)$ and $D(60)$, notes that they have opposite signs, implies that D is continuous, and invokes the Intermediate Value Theorem to conclude that $D(t)$ must equal 0 for some t in the interval.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), 1 point in part (c), no points in part (d), and the units point. In parts (a) and (b) the student's work is correct. In part (c) the student earned the first point with the substitution for $S'(7)$ in the expression for $\frac{dV}{dt}$. Prior to that step, the student was working with $\frac{dh}{dt}$ rather than $S'(t)$. The student's answer is not presented accurately to three decimal places. In part (d) the student's work is incorrect.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a), no point in part (b), 1 point in part (c), no points in part (d), and the units point. In part (a) the student has the correct limits and integrand but presents an incorrect answer of 171.183 and so earned 2 of the 3 points. In part (b) the student's decimal point is incorrectly placed. In part (c) the student establishes the relationship between V and S by connecting $\frac{dV}{dt}$ to $\frac{dh}{dt}$ and $\frac{dh}{dt}$ to S'. The student uses the truncated value 1.917 for $S'(7)$ in the computation of $\frac{dV}{dt}$, so the student's answer is incorrect. In part (d) the student's work is incorrect.

Question 2

The polar curve r is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point P on the polar curve r with x-coordinate -3. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

(c) A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

(a) Area $= \frac{1}{2}\int_{\pi/2}^{\pi} (r(\theta))^2\, d\theta = 47.513$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$
(b) $-3 = r(\theta)\cos\theta = (3\theta + \sin\theta)\cos\theta$ $\theta = 2.01692$ $y = r(\theta)\sin(\theta) = 6.272$	$3 : \begin{cases} 1 : \text{equation} \\ 1 : \text{value of } \theta \\ 1 : y\text{-coordinate} \end{cases}$
(c) $y = r(\theta)\sin\theta = (3\theta + \sin\theta)\sin\theta$ $\left.\frac{dy}{dt}\right\rvert_{\theta=2\pi/3} = \left[\frac{dy}{d\theta}\cdot\frac{d\theta}{dt}\right]_{\theta=2\pi/3} = -2.819$ The y-coordinate of the particle is decreasing at a rate of 2.819.	$3 : \begin{cases} 1 : \text{uses chain rule} \\ 1 : \text{answer} \\ 1 : \text{interpretation} \end{cases}$

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Work for problem 2(a)

$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} r^2\,d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} (3\theta + \sin\theta)^2\,d\theta$$

$$= 47.513$$

Work for problem 2(b)

$x = r\cos\theta$

$x = (3\theta + \sin\theta)\cos\theta$

$-3 = (3\theta + \sin\theta)\cos\theta$

$\theta = 2.017$ radians

The y-coordinate of point $P = r\sin\theta$

$= (3\theta + \sin\theta)\sin\theta$

$= 6.272$

Continue problem 2 on page 7.

Work for problem 2(c)

$$\frac{dy}{dt} = \frac{d(r\sin\theta)}{dt}$$

$$= \frac{d(r\sin\theta)}{d\theta} \cdot \frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[(3\theta + \sin\theta)\cdot\sin\theta\right] \cdot \frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}(3\theta\sin\theta + \sin^2\theta)\cdot\frac{d\theta}{dt}$$

$$= (3\sin\theta + 3\theta\cos\theta + \sin 2\theta)\cdot\frac{d\theta}{dt}$$

$$\left.\frac{dy}{dt}\right|_{\theta=\frac{2\pi}{3}} = \left(3\sin\frac{2\pi}{3} + 3\left(\frac{2\pi}{3}\right)\cos\frac{2\pi}{3} + \sin\frac{4\pi}{3}\right)\cdot 2$$

$$= -2.819$$

$\because$ y is positive at the instant $\theta = \frac{2\pi}{3}$ and $\frac{dy}{d\theta}$ is negative at the instant $\theta = \frac{2\pi}{3}$,

$\therefore$ the particle is travelling towards the x-axis at the instant $\theta = \frac{2\pi}{3}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Work for problem 2(a)

when the graph is in the II quadrant, $\theta \in (\frac{\pi}{2}, \pi)$

so area $= \int_{\frac{\pi}{2}}^{\pi} r(\theta)^2 \cdot d\theta = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}(3\theta + \sin\theta)^2 \cdot d\theta$

when we include the axes,

$= 49.513$

Work for problem 2(b)

$r_{(\theta)} = 3\theta + \sin\theta.$

thus $x(\theta) = (3\theta + \sin\theta) \cdot \cos\theta.$

$y(\theta) = (3\theta + \sin\theta) \cdot \sin\theta.$

when $x(\theta) = -3$, $\theta \in [\frac{\pi}{2}, \pi]$, $\theta = 2.017$

$y_{(\theta)} = x(\theta) \cdot \tan\theta = -3 \cdot \tan\theta = 6.271$

so: $\theta = 2.017$

$y(\theta) = 6.271$

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Continue problem 2 on page 7.

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Work for problem 2(c)

$\frac{dy}{dt} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dt}$

$\frac{dy}{d\theta} = \frac{d(3\theta + \sin\theta) \cdot \sin\theta}{d\theta} = 3\theta \cdot \cos\theta + 3\sin\theta + 2\sin\theta \cdot \cos\theta$

so when $\theta = \frac{2\pi}{3}$ $\frac{dy}{dt} = 2 \cdot \left[3 \cdot \frac{2\pi}{3} \cdot \left(-\frac{1}{2}\right) + 3 \cdot \frac{\sqrt{3}}{2} + 2 \cdot \frac{\sqrt{3}}{2} \cdot \left(-\frac{1}{2}\right)\right]$

$= 2\sqrt{3} - 2\pi$.

① So when $\theta = \frac{2\pi}{3}$ $\frac{dy}{dt} = 2\sqrt{3} - 2\pi$.

② it means, when $\theta = \frac{2\pi}{3}$, the speed of the particle on the direction $\vec{y}$ is $(2\sqrt{3} - 2\pi)$.

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 2(a)

$A = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2} r^2 d\theta = \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} (9\theta^2 + \sin^2\theta + 6\theta\sin\theta)\, d\theta$

$= \frac{1}{2}(3\theta^3 +$

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Work for problem 2(b)

$x = r\cos\theta$

$y = r\sin\theta$

$x = -3,\quad r\cos\theta = -3 \quad, r = 3\theta + \sin\theta$

$3\theta\cos\theta + \sin\theta\cos\theta = -3$

$y = \theta\, x \tan\theta$

Continue problem 2 on page 7.

Work for problem 2(c)

$$\frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$$

$$\cancel{= \frac{2}{3}\pi \times 2 = \frac{4}{3}\pi}$$

$$= (\sin\theta + r\cos\theta) \times 2$$

$$= \left(\frac{\sqrt{3}}{2} - \frac{1}{2}r\right) \times 2$$

$$= \sqrt{3} - r$$

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

AP® CALCULUS BC
2011 SCORING COMMENTARY (Form B)

Question 2

Sample: 2A
Score: 9

The student earned all 9 points. Because the particle is above the x-axis, it is sufficient that the student states "the particle is travelling towards the x-axis" in part (c).

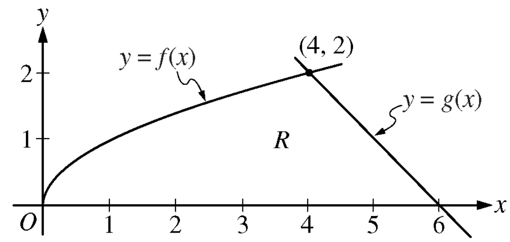
Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student's integral is correct, so the first 2 points were earned. The answer is incorrect. In part (b) the student earned the equation point implicitly and earned the point for the value of θ. The student's answer is incorrect, possibly as a result of intermediate rounding. In part (c) the student earned the first 2 points. The student does not indicate that the y-coordinate of the particle is decreasing.

Sample: 2C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student's integral is correct, so the first 2 points were earned. In part (b) the fourth line of the student's solution earned the first point. In part (c) the student earned the chain-rule point.

Question 3



The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

(a) Area $= \int_0^4 \sqrt{x}\,dx + \frac{1}{2} \cdot 2 \cdot 2 = \frac{2}{3}x^{3/2}\Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $y = \sqrt{x} \;\Rightarrow\; x = y^2$
$y = 6 - x \;\Rightarrow\; x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \;\Rightarrow\; x = \frac{1}{4}$

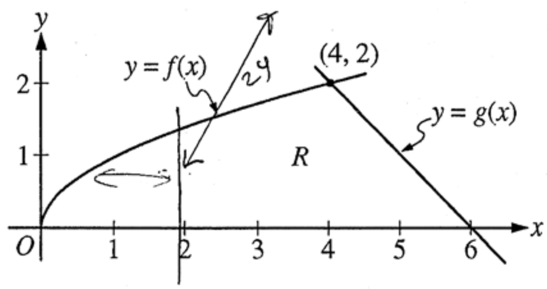
The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

$$R = \int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx =$$

$$= \frac{2}{3}x^{3/2}\Big|_0^4 + \left(6x - \frac{x^2}{2}\right)\Big|_4^6 = \frac{16}{3} + 36 - 18 - 24 + 8 =$$

$$= 2 + \frac{16}{3} = \frac{22}{3}$$

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Continue problem 3 on page 9.

Work for problem 3(b) ~~Vtt~~ for $0 \le y \le 2$: ~~R is between~~

~~$y=f(x)$ & the horizontal axis;~~

$y = \cancel{3}\sqrt{x} \Leftrightarrow x = y^2$; $\begin{cases} x=0 \\ y=0 \end{cases}$; $\begin{cases} x=\cancel{6}4 \\ y=2 \end{cases}$;

~~$V = \int_0^2$~~ $y = 6 - x \Leftrightarrow x = 6 - y \Rightarrow$

$$V = \int_0^2 \left((6 - y - y^2) \times 2y \right) dy$$

Work for problem 3(c) tangent line to f:

$y_t = f(x_0) + f'(x_0)(x - x_0)$

$y_t = \sqrt{x_0} + \frac{1}{2\sqrt{x_0}}(x - x_0)$, y_t is $\perp$ to $g(x) \Rightarrow$

$\frac{1}{2\sqrt{x_0}} = -\frac{1}{g'(x_0)}$, $g'(x_0) = (6 - x_0)' = -1 \Rightarrow$

$\frac{1}{2\sqrt{x_0}} = 1 \Leftrightarrow 2\sqrt{x_0} = 1$

$\sqrt{x_0} = \frac{1}{2}$

$x_0 = \frac{1}{4}$, $y_0 = \sqrt{x_0} = \frac{1}{2} \Rightarrow$

$P(0.25, 0.5)$

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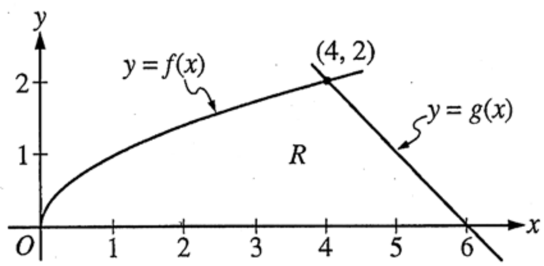
NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

a) $A = \int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= \frac{2}{3}x^{3/2}\Big]_0^4 + \left(6x - \frac{x^2}{2}\right)\Big]_4^6 = \frac{2}{3}(4)^{3/2} + \left[\left((6\times 6) - \frac{36}{2}\right) - (24-8)\right]$

$= \frac{2}{3}(2)(4) + (18-16) = \frac{16}{3} + 2 = \frac{16}{3} + \frac{6}{3} = \frac{22}{3}(\text{unit})^2.$

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Continue problem 3 on page 9.

NO CALCULATOR ALLOWED

Work for problem 3(b)

$h = 2y$

$y = 6 - x \Rightarrow x = 6 - y$

$y = \sqrt{x} \Rightarrow x = y^2$

$V = \int_a^b A(x)\,dx = \int_0^2 \left([(6-y) - y^2] \times 2[(6-y) - y^2] \right) dy$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}}$ $g'(x) = -1$

$\frac{1}{2\sqrt{x}} = 1$ $1 = 2\sqrt{x}$

$\sqrt{x} = 1/2 \Rightarrow x = \frac{1}{4}$

$f(1/4) = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow P\left(\frac{1}{4}, \frac{1}{2}\right)$

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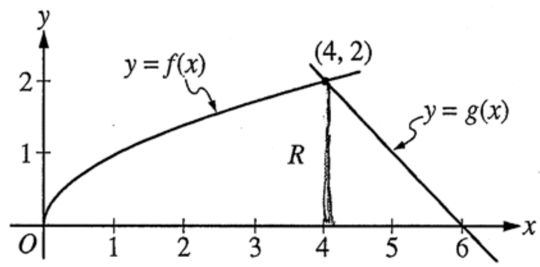
NO CALCULATOR ALLOWED

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



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Work for problem 3(a)

(a). $R = \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx$

$= 2x^{\frac{3}{2}}\Big|_0^4 + \left(6x - \frac{x^2}{2}\right)\Big|_4^6$

$= 2 \times 4^{\frac{3}{2}} - 0 + 6\times 6 - \frac{36}{2} - 6\times 4 + \frac{16}{2}$

$= 16 + 36 - 18 - 24 + 8$

$= 8$

Continue problem 3 on page 9.

NO CALCULATOR ALLOWED

Work for problem 3(b)

$V = \int_0^6 (6 - x - \sqrt{x})\,dy \quad (0 \le y \le 2)$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}} \qquad g'(x) = -1$

Since the line is perpendicular to $g(x)$, when $g'(x) = -1$.

the slope of the line is 1

Therefore $\frac{1}{2\sqrt{x}} = 1 \qquad x = \frac{1}{4}$

So $f(\frac{1}{4}) = \frac{1}{2}$ the coordinates of point P is $(\frac{1}{4}, \frac{1}{2})$

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Question 3

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

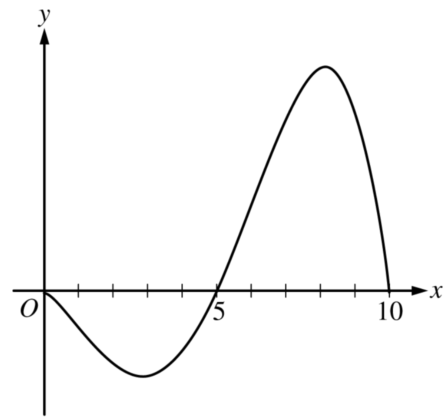
The student earned 6 points: 3 points in part (a), no points in part (b), and 3 points in part (c). In parts (a) and (c) the student's work is correct. In part (b) the student's integrand was not eligible for any points.

Sample: 3C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), and 3 points in part (c). In part (a) the student earned the integral point. The student's antidifferentiation is incorrect. The student was eligible for the answer point, but the work contains an arithmetic error. In part (b) the student's integrand is incorrect and so was not eligible for any points. In part (c) the student's work is correct.

Question 4

The graph of the differentiable function $y = f(x)$ with domain $0 \le x \le 10$ is shown in the figure above. The area of the region enclosed between the graph of f and the x-axis for $0 \le x \le 5$ is 10, and the area of the region enclosed between the graph of f and the x-axis for $5 \le x \le 10$ is 27. The arc length for the portion of the graph of f between $x = 0$ and $x = 5$ is 11, and the arc length for the portion of the graph of f between $x = 5$ and $x = 10$ is 18. The function f has exactly two critical points that are located at $x = 3$ and $x = 8$.



Graph of f

(a) Find the average value of f on the interval $0 \le x \le 5$.

(b) Evaluate $\int_0^{10} (3f(x) + 2)\,dx$. Show the computations that lead to your answer.

(c) Let $g(x) = \int_5^x f(t)\,dt$. On what intervals, if any, is the graph of g both concave up and decreasing? Explain your reasoning.

(d) The function h is defined by $h(x) = 2f\left(\frac{x}{2}\right)$. The derivative of h is $h'(x) = f'\left(\frac{x}{2}\right)$. Find the arc length of the graph of $y = h(x)$ from $x = 0$ to $x = 20$.

(a) Average value $= \frac{1}{5}\int_0^5 f(x)\,dx = \frac{-10}{5} = -2$

1 : answer

(b) $\int_0^{10} (3f(x) + 2)\,dx = 3\left(\int_0^5 f(x)\,dx + \int_5^{10} f(x)\,dx\right) + 20$

$= 3(-10 + 27) + 20 = 71$

2 : answer

(c) $g'(x) = f(x)$
$g'(x) < 0$ on $0 < x < 5$
$g'(x)$ is increasing on $3 < x < 8$.
The graph of g is concave up and decreasing on $3 < x < 5$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{analysis} \\ 1 : \text{answer and reason} \end{cases}$

(d) Arc length $= \int_0^{20} \sqrt{1 + (h'(x))^2}\,dx = \int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2}\,dx$

Let $u = \frac{x}{2}$. Then $du = \frac{1}{2}dx$ and

$$\int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2}\,dx = 2\int_0^{10} \sqrt{1 + (f'(u))^2}\,du = 2(11 + 18) = 58$$

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{substitution} \\ 1 : \text{answer} \end{cases}$

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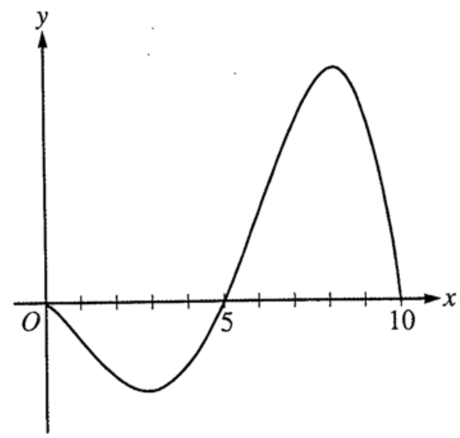

Graph of f

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Work for problem 4(a)

$$\frac{1}{5-0}\int_0^5 f(x)\,dx = \frac{1}{5-0}(-10) = \frac{-10}{5} = -2$$

-2

Work for problem 4(b)

$$\int_0^{10}(3f(x)+2)\,dx = 3\int_0^{10} f(x) + \int_0^{10} 2\,dx$$

$$= 3(-10+27) + [2x]_0^{10}$$

$$= 3(17) + 20 = 51 + 20$$
$$= 71$$

71

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

$g(x) = \int_5^x f(t)\,dt \quad \therefore g'(x) = f(x)$

graph of g is decreasing on $0 < x < 5$ because $g'(x) < 0$

graph of g is concave up on $3 < x < 8$ because $g''(x) > 0$

graph of g is both concave up and decreasing on $3 < x < 5$ (because $g'(x) < 0$ and $g''(x) > 0$)

$3 < x < 5$

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Work for problem 4(d)

$\int_0^{20} \sqrt{1 + \left(\frac{dh}{dx}\right)^2} \cdot dx = \int_0^{20} \sqrt{1 + \left(f'\left(\frac{x}{2}\right)\right)^2}\, dx = A$

$\frac{x}{2} = t \quad \frac{dt}{dx} = \frac{1}{2} \quad dt = \frac{1}{2} dx \qquad A = 2\int_0^{10} \sqrt{1 + f'(t)^2}\, dt$

$\int_0^{10} \sqrt{1 + (f'(x))^2}\, dx = 11 + 18 = 29$

$A = 2 \times 29 = 58$

58

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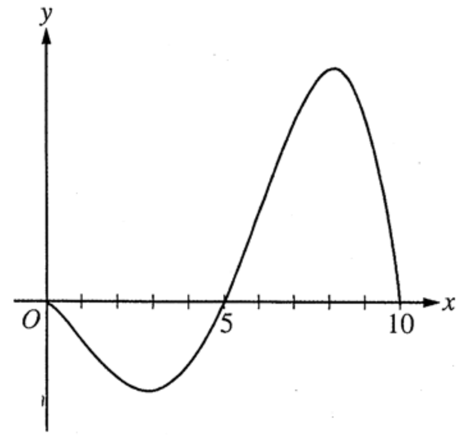

Graph of f

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Work for problem 4(a)

$$\frac{\int_0^5 f(x)\,dx}{5-0} = 2$$

Work for problem 4(b)

$$\int_0^{10} (3f(x)+2)\,dx = 3\int_0^{10} f(x)\,dx + \int_0^{10} 2\,dx = 3\times 37 + 20 = 131$$

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

To be concave up and decreasing, $g'(x)<0$, $g''(x)>0$

$g'(x) = f(x) < 0$

~~$\therefore 0<x<5$~~ $\therefore$ ~~$0<x<3,\ 8<x<10$~~ $\therefore 0<x<5$

$g''(x) = f'(x) > 0$

~~$\therefore 3<x<8$~~ ~~$\therefore 3<$~~ $\therefore 3<x<8$

$\therefore 3<x<5$

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Work for problem 4(d)

$$\int_0^{20} \sqrt{1+\{f'(x)\}^2}\,dx$$

$$= \int_0^{10} \sqrt{1+\{f'(x)\}^2}\,dx$$

$$= \int_0^{5} \sqrt{1+\{f'(x)\}^2}\,dx + \int_5^{10} \sqrt{1+\{f'(x)\}^2}\,dx = 11+18 = 29$$

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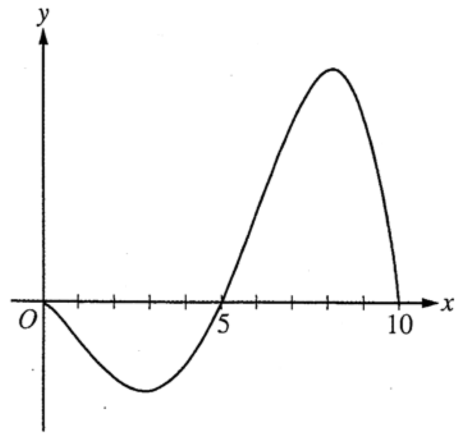

Graph of f

Work for problem 4(a)

$\frac{1}{5-0}\int_0^5 f(x)\,dx$

$= \frac{1}{5} \times 10 = 2$

Work for problem 4(b)

$\int_0^{10} (3f(x)+2)\,dx = 3\int_0^{10} f(x)\,dx + \int_0^{10} 2\,dx$

$= 17 \times 3 + 20$

$= 51 + 20$

$= 71$

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Continue problem 4 on page 11.

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Work for problem 4(c)

$g(x) = \int_5^x f(t)dt$

$g(x) = F(x) - F(5)$

$g'(x) = f(x) - f(5) \ni g'(x) > 0 \rightarrow$ g(x) is increasig for any x at x>5

$g''(x) = f'(x) - f'(5)$

Thus, there is no interval that the graph of g both concave up and decreasing.

Work for problem 4(d)

since $h(x) = 2f\left(\frac{x}{2}\right)$

arc length of h(20) is 2f(10).

thus,

arc legth of h(20) is $= 2(11+18)$

$=58$

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Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: no point in part (a), 1 point in part (b), 3 points in part (c), and 2 points in part (d). In parts (a) and (b) the student does not work correctly with the region below the x-axis. The student's work earned 1 of the 2 points in part (b). In part (c) the student's work is correct. In part (d) the student may be attempting a substitution because there are new correct limits of integration, but the factor of 2 is missing, so only 2 of the 3 possible points were earned.

Sample: 4C
Score: 3

The student earned 3 points: no point in part (a), 2 points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student does not work correctly with the region below the x-axis. In part (b) the student's work is correct. In part (c) the student's work is incorrect. In part (d) the student has the correct answer, which is achieved by doubling the arc length of the graph of $y = f(x)$ from $x = 0$ to $x = 10$ but gives no indication as to why that method works. The student earned 1 of the 3 possible points.

Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

(a) $a(5) \approx \dfrac{v(10) - v(0)}{10 - 0} = \dfrac{0.3}{10} = 0.03$ meters/sec^2

1 : answer

(b) $\int_0^{60} |v(t)|\, dt$ is the total distance, in meters, Ben rides over the 60-second interval $t = 0$ to $t = 60$.

$$\int_0^{60} |v(t)|\, dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139 \text{ meters}$$

2 : { 1 : meaning of integral; 1 : approximation }

(c) Because $\dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t, $40 < t < 60$, such that $v(t) = 2$.

2 : { 1 : difference quotient; 1 : conclusion with justification }

(d) $2L(t)L'(t) = 2B(t)B'(t)$

$$L'(40) = \frac{B(40)v(40)}{L(40)} = \frac{9 \cdot 2.5}{\sqrt{144 + 81}} = \frac{3}{2} \text{ meters/sec}$$

3 : { 1 : derivatives; 1 : uses $B'(t) = v(t)$; 1 : answer }

1 : units in (a) or (b)

NO CALCULATOR ALLOWED

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

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Work for problem 5(a)

$a(5) = \frac{V(10) - V(0)}{10-0} = \frac{2{,}3 - 2{,}0}{10} = 0{,}03$ meters/(second)^2

Work for problem 5(b)

$\int_0^{60} |v(t)|dt$ means a total distance, travelled by Ben during the time = 60 seconds, $0<t<60$.

$\int_0^{60} |v(t)|dt = v(0)\cdot\Delta t + v(10)\,\Delta t + v(40)\Delta t =$

$= 2\cdot 10 + 2{,}3\cdot 30 + 2{,}5\cdot 20 = 139$ meters

Continue problem 5 on page 13.

Work for problem 5(c)

Accoarding to Mean Value Teozem

$$\frac{B(60)-B(40)}{60-40} = \frac{49-9}{20} = 2 \ ;$$

$\Rightarrow$ There must be a time t when velecity is equal to 2 meters/second

Work for problem 5(d)

$$I\varsigma \quad (L(t))^2 = 12^2 + (B(t))^2$$

$$L(t) = \sqrt{12^2+(B(t))^2}$$

$$L'(t) = \frac{2B(t)\cdot B'(t)}{2\sqrt{12^2+(B(t))^2}}$$

$$L'(40) = \frac{9\cdot 2{,}5}{\sqrt{225}} = \frac{9\cdot 2{,}5}{15} = \frac{3\cdot 2{,}5}{5} = \frac{7{,}5}{5} =$$

$= 1{,}5$ meters/second

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NO CALCULATOR ALLOWED

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Work for problem 5(a)

$a(5) = v'(5) = \frac{v(10)-v(0)}{10-0} = \frac{2.3-2}{10} = 0.03$

Work for problem 5(b)

the meaning of $\int_0^{60} |v(t)|\,dt$: The total distance Ben rides from $t=0$ to $t=60$, which is measured by meters.

Left Riemann Sum approximation of $\int_0^{60} |v(t)|\,dt$:

$\int_0^{60} |v(t)|\,dt = 2\times 10 + 2.3\times(40-10) + 2.5\times(60-40)$
$= 20 + 69 + 50$
$= 139$

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

uncertain. the total change from t=40 to t=60 is increase. If the velocity is 2 meters per second, there will be a decrease in $40 \le t \le 60$. We can't find the exact change between $40 \le t \le 60$, so there is uncertainly a velocity is 2 meters per second

Work for problem 5(d)

$L(t)^2 = 12^2 + B(t)^2$ when t=40

$2L(40)\frac{dL}{dt} = 144 + 2B(40)\frac{dB}{dt}$ and $L(40) = \sqrt{144+B(40)^2}$

$= \sqrt{144+81}$

$= \sqrt{225}$

$= 15$

$2\times15\frac{dL}{dt} = 144 + 2\times9\frac{dB}{dt}$

$\frac{dL}{dt} = \frac{144+45}{30}$

$= 63$ meters per second

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NO CALCULATOR ALLOWED

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Work for problem 5(a)

$a(t)$ at $t=5$: $\dfrac{v(10)-v(0)}{10-0}$

$= \dfrac{2.3-2.0}{10}$

$= \dfrac{.3}{10}$

$\boxed{= .03 \text{ m}^2/\text{second}}$

Work for problem 5(b)

$\int_0^{60} |v(t)|\,dt$ would be the total distance covered by Ben, whether going forwards or backwards.

$$\begin{array}{r} 136 \\ \underline{\times\ 30} \\ 4080 \end{array}$$

$\int_0^{60} |v(t)|\,dt \approx 10(100) + 30(136) + 20(9)$

$= 1000 + 4080 + 180$

$= \boxed{5260 \text{ meters}}$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$= \frac{49-9}{60-40}$

$= \frac{40}{20}$

$= 2$ m/s

According to the mean value theorem, there is a time $t=c$ in which Ben's velocity is 2 meters per second.

Work for problem 5(d)

$(L(t))^2 = 12^2 + (B(t))^2$

$(L(t))^2 = 144 + (B(40))^2$

$(L(t))^2 = 144 + 9^2$

$\sqrt{(L(t))^2} = \sqrt{225}$

$L(t) = 15$

$\frac{d}{dx}$

$$\begin{array}{r} 144 \\ +\ 81 \\ \hline 225 \end{array}$$

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points. In part (d) the student solves for $L(t)$ prior to differentiating.

Sample: 5B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), no points in part (c), 2 points in part (d), and the units point. In parts (a) and (b) the student's work is correct. In part (c) the student's work is incorrect. In part (d) the student differentiates the expression incorrectly. The student uses $B'(40) = v(40) = 2.5,$ so the second point was earned. Because the student's derivative is of the form $LL' = BB' + C,$ where C is a constant, the student was eligible for the answer point. The student's answer is consistent with the expression presented, so the answer point was earned. The student earned the units point because "meters" are mentioned in the interpretation of the meaning of the integral in part (b).

Sample: 5C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), 2 points in part (c), no points in part (d), and the units point. In parts (a) and (c) the student's work is correct. In part (b) the student does not mention the time interval, and the approximation is incorrect. In part (d) the student's work is incorrect. The student presents the correct units in part (b), so the units point was earned. The units in part (a) are incorrect.

Question 6

Let $f(x) = \ln\left(1 + x^3\right)$.

(a) The Maclaurin series for $\ln(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \cdot \frac{x^n}{n} + \cdots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for f.

(b) The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer.

(c) Write the first four nonzero terms of the Maclaurin series for $f'\left(t^2\right)$. If $g(x) = \int_0^x f'\left(t^2\right) dt$, use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$.

(d) The Maclaurin series for g, evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\frac{1}{5}$.

(a) $x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots + (-1)^{n+1} \cdot \frac{x^{3n}}{n} + \cdots$	$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$
(b) The interval of convergence is centered at $x = 0$. At $x = -1$, the series is $-1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \cdots - \frac{1}{n} - \cdots$, which diverges because the harmonic series diverges. At $x = 1$, the series is $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + (-1)^{n+1} \cdot \frac{1}{n} + \cdots$, the alternating harmonic series, which converges. Therefore the interval of convergence is $-1 < x \le 1$.	2 : answer with analysis
(c) The Maclaurin series for $f'(x)$, $f'\left(t^2\right)$, and $g(x)$ are $f'(x) : \sum_{n=1}^{\infty} (-1)^{n+1} \cdot 3x^{3n-1} = 3x^2 - 3x^5 + 3x^8 - 3x^{11} + \cdots$ $f'\left(t^2\right) : \sum_{n=1}^{\infty} (-1)^{n+1} \cdot 3t^{6n-2} = 3t^4 - 3t^{10} + 3t^{16} - 3t^{22} + \cdots$ $g(x) : \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3x^{6n-1}}{6n-1} = \frac{3x^5}{5} - \frac{3x^{11}}{11} + \frac{3x^{17}}{17} - \frac{3x^{23}}{23} + \cdots$ Thus $g(1) \approx \frac{3}{5} - \frac{3}{11} = \frac{18}{55}$.	$4 : \begin{cases} 1 : \text{two terms for } f'\left(t^2\right) \\ 1 : \text{other terms for } f'\left(t^2\right) \\ 1 : \text{first two terms for } g(x) \\ 1 : \text{approximation} \end{cases}$
(d) The Maclaurin series for g evaluated at $x = 1$ is alternating, and the terms decrease in absolute value to 0. Thus $\left\lvert g(1) - \frac{18}{55} \right\rvert < \frac{3 \cdot 1^{17}}{17} = \frac{3}{17} < \frac{1}{5}$.	1 : analysis

Work for problem 6(a)

$g(x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \cdot \frac{x^n}{n} + \cdots$

$f(x) = \ln(1+x^3) = g(x^3) = x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots + (-1)^{n+1} \frac{x^{3n}}{n} + \cdots$

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Work for problem 6(b)

The series is centered around $x=0$. The interval is $-1 < x < 1$. If we check the boundaries;

$x=-1 \to -1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} \cdots$, which diverges (harmonic)

$x=1 \to 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots$, which converges (alternating harmonic)

so, the interval of convergence is $-1 < x \leq 1$

Continue problem 6 on page 15.

Work for problem 6(c)

$f'(x) = 3x^2 - 3x^5 + 3x^8 - 3x^{11}$

$f'(t^2) = 3t^4 - 3t^{10} + 3t^{16} - 3t^{22}$

$g(x) = \int_0^x f'(t^2)\,dt =$

$g(1) = \int_0^1 f'(t^2)\,dt = \int_0^1 (3t^4 - 3t^{10} + 3t^{16})\,dt$

$g(1) \simeq \left(\frac{3}{5}t^5 - \frac{3}{11}t^{11}\right)\Big|_0^1 = \frac{3}{5} - \frac{3}{11} = \frac{33-15}{55} = \frac{18}{55}$

Work for problem 6(d)

$g(x) = \left(\frac{3}{5}t^5 - \frac{3}{11}t^{11} + \frac{3}{17}t^{17}\ldots\right)\Big|_0^x$

$g(x) = \frac{3}{5}x^5 - \frac{3}{11}x^{11} + \frac{3}{17}x^{17}\ldots$

$e = \left|g(1) - \frac{18}{55}\right|$ must be smaller then the next term in the series, which is $\frac{3}{17}$, so

$\left|g(1) - \frac{18}{55}\right| < \frac{3}{17} < \frac{1}{5}$

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Work for problem 6(a)

f(x) puts x^3 instead of x on the Maclaurin series for $\ln(x+1)$ ∴

So, $x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots + (-1)^{n+1} \cdot \frac{x^{3n}}{n} + \cdots$

Work for problem 6(b)

If we use ratio test

$$\frac{\frac{(-1)^{n+2} \cdot x^{3n+3}}{n+1}}{\frac{(-1)^{n+1} \cdot x^{3n}}{n}} = \left| \frac{n x^{3n+3}}{(n+1) x^{3n}} \right| = |x^3| < 1$$

∴ $-1 < x^3 < 1$ ∴ $-1 < x < 1$

$-1 < x < 1$

Continue problem 6 on page 15.

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NO CALCULATOR ALLOWED

Work for problem 6(c)

$f(t): t^3 - \frac{t^6}{2} + \frac{t^9}{3} - \frac{t^{12}}{4} + \cdots$ $\quad f'(t): 3t^2 - 3t^5 + 3t^8 - 3t^{11} \cdots$

$f'(t^2): 3t^4 - 3t^{10} + 3t^{16} - 3t^{22} + \cdots$

$g(1) = \int_0^1 f'(t^2)\,dt = \left(\frac{3t^5}{5} - \frac{3t^{11}}{11} + \cdots\right)\Big|_0^1$

$= \frac{3}{5} - \frac{3}{11} = \frac{18}{55}$

$\therefore \frac{18}{55}$

Work for problem 6(d)

I predicted g(1) by using first two nonzero terms.

However the third term is $\frac{3t^{17}}{17} \to \frac{3}{17}$

$\frac{3}{17} = 0.176 \cdots < \frac{1}{5}$

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NO CALCULATOR ALLOWED

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Work for problem 6(a)

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

$\therefore f(x) = \ln(1+x^3) = x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots + (-1)^{n+1}\frac{x^{3n}}{n} + \cdots$

Work for problem 6(b)

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{(-1)^{n+2}\frac{x^{3(n+1)}}{n+1}}{(-1)^{n+1}\frac{x^{3n}}{n}} = (-1)\cdot x^3$

$= -x^3$

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Continue problem 6 on page 15.

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Work for problem 6(c)

$f'(x) = 3x^2 - \frac{6x^5}{2} + \frac{9x^8}{3} - \frac{12 \cdot x^{11}}{4} + \cdots + (-1)^{n+1} \frac{3n \cdot x^{3n-1}}{n} + \cdots$

$f'(t^2) = 3t^4 - \frac{6 \times t^{10}}{2} + \frac{9 \times x^{16}}{3} - \frac{12 \cdot x^{22}}{4} + \cdots$

$g(1) = 3 \times 1 - \frac{6 \times 1}{2}$

$= 3 - 3 = 0.$

Work for problem 6(d)

when $x = 1$.

$f(x) = 3 - 3 \cdot 1 + 3 \cdot 1 - 3 \cdot 1 + \cdots + (-1)^2 \cdot 3$

$\therefore 0 < \frac{1}{5}$ $\therefore$ it abolute value is to 0.

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AP® CALCULUS BC
2011 SCORING COMMENTARY (Form B)

Question 6

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 4 points in part (c), and no point in part (d). In parts (a) and (c) the student's work is correct. In part (b) the student's work is incorrect. In part (d) the student uses the correct approach and has correct calculations, but the student's argument is incomplete in that it does not indicate that the error (the difference between $g(1)$ and the approximation) is what is less than $\frac{3}{17}$.

Sample: 6C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 2 points in part (c), and no point in part (d). In part (a) the student's work is correct. In parts (b) and (d) the student's work is incorrect. In part (c) the student earned the first 2 points.